FYJC - MATHEMATICS & STATISTICS

PAPER - I

COMPOUND ANGLES

EX - 5.3. ADDITION FORMULAE

Compiled & Conducted @ JKSC

COMPOUND ANGLES - 5.3

ADDITION FORMULAE

Q - 1

01. $\dfrac{\sin(A+B)}{\sin(A-B)} = \dfrac{\tan A + \tan B}{\tan A - \tan B}$

02. $\dfrac{\cos(A-B)}{\cos(A+B)} = \dfrac{\cot A \cdot \cot B + 1}{\cot A \cdot \cot B - 1}$

03. $\dfrac{\sin(A-B)}{\sin A \cdot \sin B} + \dfrac{\sin(B-C)}{\sin B \cdot \sin C} + \dfrac{\sin(C-A)}{\sin C \cdot \sin A} = 0$

04. $\sin A \cdot \sin(B-C) + \sin B \cdot \sin(C-A) + \sin C \cdot \sin(A-B) = 0$

Q - 2

01. $\sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B$

02. $\sin(A+B) \cdot \sin(A-B) = \cos^2 B - \cos^2 A$

03. $\dfrac{\tan(A+B)}{\cot(A-B)} = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B}$

Q - 3

01. $\dfrac{\tan 5A - \tan 3A}{\tan 5A + \tan 3A} = \dfrac{\sin 2A}{\sin 8A}$

02. $\dfrac{\cot A \cdot \cot 4A + 1}{\cot A \cdot \cot 4A - 1} = \dfrac{\cos 3A}{\cos 5A}$

03. $\dfrac{\cot 6A - \tan 2A}{\cot 6A + \tan 2A} = \dfrac{\cos 8A}{\cos 4A}$

04. $\dfrac{\cot x \cdot \cot y - 1}{\cot x + \cot y} = \cot(x+y)$

Q - 4

01. $\tan 8\theta - \tan 5\theta - \tan 3\theta = \tan 8\theta \cdot \tan 5\theta \cdot \tan 3\theta$

02. $\tan 3\theta - \tan 2\theta - \tan \theta = \tan 3\theta \cdot \tan 2\theta \cdot \tan \theta$

03. $\tan 15^o + \tan 30^o + \tan 15^o \cdot \tan 30^o = 1$

04. $\tan 20^o + \tan 25^o + \tan 20^o \cdot \tan 25^o = 1$

05. $\tan 18^o + \tan 27^o + \tan 18^o \cdot \tan 27^o = 1$

06. $\tan 50^o = \tan 40^o + 2 \tan 10^o$

07. $\tan 65^o = \tan 25^o + 2 \tan 40^o$

08. $\tan 54^o = \tan 36^o + 2 \tan 18^o$

Q - 5

01. $\dfrac{\cos 27^o + \sin 27^o}{\cos 27^o - \sin 27^o} = \tan 72^o$

02. $\dfrac{\cos 11^o - \sin 11^o}{\cos 11^o + \sin 11^o} = \cot 56^o$

03. $\dfrac{\cos 15^o + \sin 15^o}{\cos 15^o - \sin 15^o} = \sqrt{3}$

04. $\dfrac{\cos 8^o + \sin 8^o}{\cos 8^o - \sin 8^o} = \cot 37^o$

05. $\tan\left(\dfrac{\pi}{4} + \theta\right) - \tan\left(\dfrac{\pi}{4} - \theta\right) = \dfrac{4 \tan \theta}{1 - \tan^2\theta}$

06. $\dfrac{1}{\tan 3\theta - \tan \theta} + \dfrac{1}{\cot \theta - \cot 3\theta} = \cot 2\theta$

07. $\tan A = 5/6$; $\tan B = 1/11$; prove that : $A + B = \pi/4$

Q - 6

1. $\sin(25^o + \theta).\cos(25^o - \phi) - \cos(25^o + \theta).\sin(25^o - \phi)$

$= \sin(\theta + \phi)$

2. $\cos(35^o + A) . \cos(35^o - B) + \sin(35^o + A) .\sin(35^o - B)$

$= \cos(A + B)$

3. $\sin\left(\dfrac{\pi}{6} + A\right) . \cos\left(\dfrac{\pi}{3} - B\right) + \sin\left(\dfrac{\pi}{3} - B\right) . \cos\left(\dfrac{\pi}{6} + A\right)$

$= \cos(A - B)$

4. $\cos\left(\dfrac{\pi}{6} - A\right) . \cos\left(\dfrac{\pi}{3} + B\right) - \sin\left(\dfrac{\pi}{6} - A\right) . \sin\left(\dfrac{\pi}{3} + B\right)$

$= \sin(A - B)$

Q - 7

01. $\sin A = \dfrac{4}{5}$; $\dfrac{\pi}{2} < A < \pi$; $\cos B = \dfrac{5}{13}$; $\dfrac{3\pi}{2} < B < 2\pi$

Find i) $\sin(A + B)$ ii) $\cos(A - B)$ iii) $\tan(A - B)$

ans : $56/65$; $-63/65$; $16/63$

02. $\sin A = \dfrac{-5}{13}$; $\pi < A < \dfrac{3\pi}{2}$; $\cos B = \dfrac{3}{5}$; $\dfrac{3\pi}{2} < B < 2\pi$

Find i) $\sin(A + B)$ ii) $\cos(A - B)$ iii) $\tan(A + B)$

ans : $33/65$; $-16/65$; $-33/56$

Q-1

SOLUTION SET

01. $\dfrac{\sin(A+B)}{\sin(A-B)} = \dfrac{\tan A + \tan B}{\tan A - \tan B}$

RHS

$$= \frac{\tan A + \tan B}{\tan A - \tan B}$$

$$= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}}$$

$$= \frac{\frac{\sin A.\cos B + \cos A \sin B}{\cos A . \cos B}}{\frac{\sin A.\cos B + \cos A \sin B}{\cos A . \cos B}}$$

$$= \frac{\sin(A+B)}{\sin(A-B)} \qquad = \text{LHS}$$

02. $\dfrac{\cos(A-B)}{\cos(A+B)} = \dfrac{\cot A . \cot B + 1}{\cot A . \cot B - 1}$

RHS

$$= \frac{\cot A . \cot B + 1}{\cot A . \cot B - 1}$$

$$= \frac{\frac{\cos A}{\sin A} . \frac{\cos B}{\sin B} + 1}{\frac{\cos A}{\sin A} . \frac{\cos B}{\sin B} - 1}$$

$$= \frac{\frac{\cos A . \cos B + \sin A . \sin B}{\sin A . \sin B}}{\frac{\cos A . \cos B - \sin A . \sin B}{\sin A . \sin B}}$$

$$= \frac{\cos(A-B)}{\cos(A+B)} \qquad = \text{LHS}$$

03.

$\dfrac{\sin(A-B)}{\sin A . \sin B} + \dfrac{\sin(B-C)}{\sin B . \sin C} + \dfrac{\sin(C-A)}{\sin C . \sin A} = 0$

$$\frac{\sin(A-B)}{\sin A . \sin B} = \frac{\sin A \cos B - \cos A \sin B}{\sin A . \sin B}$$

$$= \frac{\sin A \cos B}{\sin A \sin B} - \frac{\cos A \sin B}{\sin A \sin B}$$

$$= \cot B - \cot A$$

$$\frac{\sin(B-C)}{\sin B . \sin C} = \frac{\sin B \cos C - \cos B \sin C}{\sin B . \sin C}$$

$$= \frac{\sin B \cos C}{\sin B \sin C} - \frac{\cos B \sin C}{\sin B \sin C}$$

$$= \cot C - \cot B$$

$$\frac{\sin(C-A)}{\sin C . \sin A} = \frac{\sin C \cos A - \cos C \sin A}{\sin C . \sin A}$$

$$= \frac{\sin C \cos A}{\sin C \sin A} - \frac{\cos C \sin A}{\sin C \sin A}$$

$$= \cot A - \cot C$$

LHS

$$\frac{\sin(A-B)}{\sin A . \sin B} + \frac{\sin(B-C)}{\sin B . \sin C} + \frac{\sin(C-A)}{\sin C . \sin A}$$

$$= \cot B - \cot A + \cot C - \cot B + \cot A - \cot C$$

$$= 0 \qquad = \text{RHS}$$

04. $\sin A . \sin(B-C) + \sin B . \sin(C-A) + \sin C . \sin(A-B) = 0$

LHS

$\sin A . \sin(B-C)$

$= \sin A . (\sin B \cos C - \cos B \sin C)$

$= \sin A.\sin B.\cos C - \sin A.\cos B.\sin C$

$\sin B . \sin(C-A)$

$= \sin B . (\sin C \cos A - \cos C \sin A)$

$= \sin B.\sin C.\cos A - \sin B.\cos C.\sin A$

$= \cos A.\sin B.\sin C - \sin A.\sin B.\cos C$

$\sin C . \sin(A-B)$

$= \sin C . (\sin A \cos B - \cos A \sin B)$

$= \sin C.\sin A.\cos B - \sin C.\cos A.\sin B$

$= \sin A.\cos B \sin C - \cos A.\sin B \sin C$

LHS $= \sin A . \sin(B-C) + \sin B . \sin(C-A) + \sin C . \sin(A-B)$

$= \sin A.\sin B.\cos C - \sin A.\cos B.\sin C$

$+ \cos A.\sin B.\sin C - \sin A.\sin B.\cos C$

$+ \sin A.\cos B \sin C - \cos A.\sin B \sin C$

$= 0$

Q - 2

01. $\mathbf{\sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B}$

LHS

$= \sin(A+B) \cdot \sin(A-B)$

$= (\sin A.\cos B + \cos A.\sin B)(\sin A.\cos B - \cos A.\sin B)$

$= \sin^2 A.\cos^2 B - \cos^2 A.\sin^2 B$ → SINCE WE NEED ONLY SIN^2. IN RHS WE CHANGE COS^2 TO SIN^2

$= \sin^2 A.(1 - \sin^2 B) - (1 - \sin^2 A).\sin^2 B$

$= \sin^2 A - \sin^2 A.\sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$

$= \sin^2 A - \sin^2 B$

$=$ RHS

02. $\mathbf{\sin(A+B) \cdot \sin(A-B) = \cos^2 B - \cos^2 A}$

LHS

$= \sin(A+B) \cdot \sin(A-B)$

$= (\sin A.\cos B + \cos A.\sin B)(\sin A.\cos B - \cos A.\sin B)$

$= \underline{\sin^2 A}.\cos^2 B - \cos^2 A.\underline{\sin^2 B}$ → SINCE WE NEED ONLY COS^2. , IN RHS WE CHANGE SIN^2 TO COS^2

$= (1 - \cos^2 A).\cos^2 B - \cos^2 A.(1 - \cos^2 B)$

$= \cos^2 B - \cos^2 A.\cos^2 B - \cos^2 A + \cos^2 A \cos^2 B$

$= \cos^2 B - \cos^2 A$

$=$ RHS

03. $\mathbf{\dfrac{\tan(A+B)}{\cot(A-B)} = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B}}$

LHS

$= \tan(A+B) \cdot \tan(A-B)$

$= \dfrac{\sin(A+B)}{\cos(A+B)} \cdot \dfrac{\sin(A-B)}{\cos(A-B)}$

$= \dfrac{(\sin A.\cos B + \cos A.\sin B)(\sin A.\cos B - \cos A.\sin B)}{(\cos A.\cos B - \sin A.\sin B)(\cos A.\cos B + \sin A.\sin B)}$

$= \dfrac{\sin^2 A\,.\mathbf{\cos^2 B} - \mathbf{\cos^2 A}.\sin^2 B}{\cos^2 A.\mathbf{\cos^2 B} - \mathbf{\sin^2 A}.\sin^2 B}$ → SINCE RHS HAS ONLY SIN^2. IN THE NUMERATOR WE CHANGE COS^2 TO SIN^2

$$= \frac{\sin^2 A.(1 - \sin^2 B) - (1 - \sin^2 A).\sin^2 B}{\cos^2 A.(1 - \sin^2 B) - (1 - \cos^2 A).\sin^2 B}$$

SINCE RHS HAS COS^{2}A & SIN^{2}BIN THE DENOMINBATOR , WE CHANGE SIN^{2}A TO COS^{2}A & COS^{2}B TO SIN^{2}B

$$= \frac{\sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A.\sin^2 B}{\cos^2 A - \cos^2 A \sin^2 B - \sin^2 B + \cos^2 A.\sin^2 B}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B}$$

Q - 3

01. $\dfrac{\tan 5A - \tan 3A}{\tan 5A + \tan 3A} = \dfrac{\sin 2A}{\sin 8A}$

LHS

$$= \frac{\tan 5A - \tan 3A}{\tan 5A + \tan 3A}$$

$$= \frac{\dfrac{\sin 5A}{\cos 5A} - \dfrac{\sin 3A}{\cos 3A}}{\dfrac{\sin 5A}{\cos 5A} + \dfrac{\sin 3A}{\cos 3A}}$$

$$= \frac{\dfrac{\sin 5A . \cos 3A - \cos 5A . \sin 3A}{\cos 5A . \cos 3A}}{\dfrac{\sin 5A . \cos 3A + \cos 5A . \sin 3A}{\cos 5A . \cos 3A}}$$

$$= \frac{\sin (5A - 3A)}{\sin (5A + 3A)}$$

$$= \frac{\sin 2A}{\sin 8A} \qquad = \text{RHS}$$

02. $\dfrac{\cot A . \cot 4A + 1}{\cot A . \cot 4A - 1} = \dfrac{\cos 3A}{\cos 5A}$

LHS

$$= \frac{\cot 4A . \cot A + 1}{\cot 4A . \cot A - 1}$$

$$= \frac{\dfrac{\cos 4A . \cos A}{\sin 4A \ \sin A} + 1}{\dfrac{\cos 4A . \cos A}{\sin 4A \ \sin A} - 1}$$

$$= \frac{\dfrac{\cos 4A . \cos A + \sin 4A \ \sin A}{\sin 4A . \sin A}}{\dfrac{\cos 4A . \cos A - \sin 4A \ \sin A}{\sin 4A . \sin A}}$$

$$= \frac{\cos (4A - A)}{\cos (4A + A)}$$

$$= \frac{\cos 3A}{\cos 5A} \qquad = \text{RHS}$$

03 . $\dfrac{\cot 6A - \tan 2A}{\cot 6A + \tan 2A} = \dfrac{\cos 8A}{\cos 4A}$

LHS

$$= \frac{\cot 6A - \tan 2A}{\cot 6A + \tan 2A}$$

$$= \frac{\dfrac{\cos 6A}{\sin 6A} - \dfrac{\sin 2A}{\cos 2A}}{\dfrac{\cos 6A}{\sin 6A} + \dfrac{\sin 2A}{\cos 2A}}$$

$$= \frac{\dfrac{\cos 6A . \cos 2A - \sin 6A . \sin 2A}{\sin 6A . \cos 2A}}{\dfrac{\cos 6A . \cos 2A + \sin 6A . \sin 2A}{\sin 6A . \cos 2A}}$$

$$= \frac{\cos (6A + 2A)}{\cos (6A - 2A)}$$

$$= \frac{\cos 8A}{\cos 4A} \qquad = \text{RHS}$$

04. $\dfrac{\cot x \cdot \cot y - 1}{\cot x + \cot y} = \cot(x+y)$

LHS

$= \dfrac{\cot x \cdot \cot y - 1}{\cot x + \cot y}$

$= \dfrac{\dfrac{\cos x}{\sin x} \cdot \dfrac{\cos y}{\sin y} - 1}{\dfrac{\cos x}{\sin x} + \dfrac{\cos y}{\sin y}}$

$= \dfrac{\dfrac{\cos x \cdot \cos y - \sin x \cdot \sin y}{\sin x \cdot \sin y}}{\dfrac{\cos x \cdot \sin y + \sin x \cdot \cos y}{\sin x \cdot \sin y}}$

$= \dfrac{\cos(x+y)}{\sin(x+y)}$

$= \cot(x+y)$

Q-4

01.

$\tan 3\theta - \tan 2\theta - \tan\theta = \tan 3\theta \cdot \tan 2\theta \cdot \tan\theta$

$\tan 3\theta = \tan(2\theta + \theta)$

$\tan 3\theta = \dfrac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \cdot \tan\theta}$

$\tan 3\theta - \tan 3\theta \cdot \tan 2\theta \cdot \tan\theta = \tan 2\theta + \tan\theta$

$\tan 3\theta - \tan 2\theta - \tan\theta = \tan 3\theta \cdot \tan 2\theta \cdot \tan\theta$

……. PROVED

02.

$\tan 8\theta - \tan 5\theta - \tan 3\theta = \tan 8\theta \cdot \tan 5\theta \cdot \tan 3\theta$

$\tan 8\theta = \tan(5\theta + 3\theta)$

$\tan 8\theta = \dfrac{\tan 5\theta + \tan 3\theta}{1 - \tan 5\theta \cdot \tan 5\theta}$

$\tan 8\theta - \tan 8\theta \cdot \tan 5\theta \cdot \tan 3\theta = \tan 5\theta + \tan 3\theta$

$\tan 8\theta - \tan 5\theta - \tan 3\theta = \tan 8\theta \cdot \tan 5\theta \cdot \tan 3\theta$

……. PROVED

03.

$\tan 15^\circ + \tan 30^\circ + \tan 15^\circ \cdot \tan 30^\circ = 1$

$\tan 45^\circ = \tan(15 + 30)$

$1 = \dfrac{\tan 15^\circ + \tan 30^\circ}{1 - \tan 15^\circ \cdot \tan 30^\circ}$

$1 - \tan 15^\circ \cdot \tan 30^\circ = \tan 15^\circ + \tan 30^\circ$

$1 = \tan 15^\circ + \tan 30^\circ + \tan 15^\circ \cdot \tan 30^\circ$

04.

$\tan 20^\circ + \tan 25^\circ + \tan 20^\circ \cdot \tan 25^\circ = 1$

$\tan 45^\circ = \tan(20 + 25)$

$1 = \dfrac{\tan 20^\circ + \tan 25^\circ}{1 - \tan 20^\circ \cdot \tan 25^\circ}$

$1 - \tan 20^\circ \cdot \tan 25^\circ = \tan 20^\circ + \tan 25^\circ$

$1 = \tan 25^\circ + \tan 25^\circ + \tan 20^\circ \cdot \tan 25^\circ$

05.

$\tan 18^\circ + \tan 27^\circ + \tan 18^\circ \cdot \tan 27^\circ = 1$

$\tan 45^\circ = \tan(18 + 27)$

$1 = \dfrac{\tan 18^\circ + \tan 27^\circ}{1 - \tan 18^\circ \cdot \tan 27^\circ}$

$1 - \tan 18^\circ \cdot \tan 27^\circ = \tan 18^\circ + \tan 27^\circ$

$1 = \tan 18^\circ + \tan 27^\circ + \tan 18^\circ \cdot \tan 27^\circ$

06. **$\tan 50^\circ = \tan 40^\circ + 2\tan 10^\circ$**

$\tan 50^\circ = \tan(40 + 10)$

$\tan 50^\circ = \dfrac{\tan 40^\circ + \tan 10^\circ}{1 - \tan 40^\circ \cdot \tan 10^\circ}$

$\tan 50 - \tan 50 \cdot \tan 40 \cdot \tan 10 = \tan 40 + \tan 10$

$\tan 50 - \tan(90-40) \cdot \tan 40 \cdot \tan 10 = \tan 40 + \tan 10$

$\tan 50 - \cot 40 \cdot \tan 40 \cdot \tan 10 = \tan 40 + \tan 10$

$\tan 50^\circ - \tan 10^\circ = \tan 40^\circ. + \tan 10^\circ$

$\tan 50^\circ = \tan 40^\circ + 2\tan 10^\circ.$

07. $\tan 65^\circ = \tan 25^\circ + 2\tan 40^\circ$

$\tan 65^\circ = \tan(25+40)$

$\tan 65^\circ = \dfrac{\tan 25^\circ + \tan 40^\circ}{1 - \tan 25^\circ.\tan 40^\circ}$

$\tan 65 - \tan 65.\tan 25.\tan 40 = \tan 25 + \tan 40$

$\tan 65 - \tan(90-25).\tan 25.\tan 40 = \tan 25 + \tan 40$

$\tan 65 - \cot 25.\tan 25.\tan 40 = \tan 25 + \tan 40$

$\tan 65^\circ - \tan 40^\circ = \tan 25^\circ. + \tan 40^\circ$

$\tan 65^\circ = \tan 25^\circ + 2\tan 40^\circ.$

08. $\tan 54^\circ = \tan 36^\circ + 2\tan 18^\circ$

$\tan 54^\circ = \tan(36+18)$

$\tan 54^\circ = \dfrac{\tan 36^\circ + \tan 18^\circ}{1 - \tan 36^\circ.\tan 18^\circ}$

$\tan 54 - \tan 54.\tan 36.\tan 18 = \tan 36 + \tan 18$

$\tan 54 - \tan(90-36).\tan 36.\tan 18 = \tan 36 + \tan 18$

$\tan 54 - \cot 36 . \tan 36 . \tan 18 = \tan 36 + \tan 18$

$\tan 54^\circ - \tan 18^\circ = \tan 36^\circ + \tan 18^\circ$

$\tan 54^\circ = \tan 36^\circ + 2\tan 18^\circ.$

Q-5

01. $\dfrac{\cos 27^\circ + \sin 27^\circ}{\cos 27^\circ - \sin 27^\circ} = \tan 72^\circ$

LHS

$\tan 72^\circ = \tan(45+27)$

$= \dfrac{\tan 45^\circ + \tan 27^\circ}{1 - \tan 45^\circ.\tan 27^\circ}$

$= \dfrac{1 + \tan 27^\circ}{1 - \tan 27^\circ}$

$= \dfrac{1 + \dfrac{\sin 27^\circ}{\cos 27^\circ}}{1 - \dfrac{\sin 27^\circ}{\cos 27^\circ}}$

$= \dfrac{\cos 27^\circ + \sin 27^\circ}{\cos 27^\circ - \sin 27^\circ}$ = RHS

02. $\dfrac{\cos 11^\circ - \sin 11^\circ}{\cos 11^\circ + \sin 11^\circ} = \cot 56^\circ$

$\tan 56^\circ = \tan(45+11)$

$= \dfrac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ.\tan 11^\circ}$

$\tan 56^\circ = \dfrac{1 + \tan 11^\circ}{1 - \tan 11^\circ}$

$\tan 56^\circ = \dfrac{1 + \dfrac{\sin 11^\circ}{\cos 11^\circ}}{1 - \dfrac{\sin 11^\circ}{\cos 11^\circ}}$

$\tan 56^\circ = \dfrac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$

$\cot 56^\circ = \dfrac{\cos 11^\circ - \sin 11^\circ}{\cos 11^\circ + \sin 11^\circ}$

03. $\dfrac{\cos 8^\circ + \sin 8^\circ}{\cos 8^\circ - \sin 8^\circ} = \cot 37^\circ$

$\tan 37^\circ = \tan(45-8)$

$= \dfrac{\tan 45^\circ - \tan 8^\circ}{1 + \tan 45^\circ.\tan 8^\circ}$

$= \dfrac{1 - \tan 8^\circ}{1 + \tan 8^\circ}$

$= \dfrac{1 - \dfrac{\sin 8^\circ}{\cos 8^\circ}}{1 + \dfrac{\sin 8^\circ}{\cos 8^\circ}}$

$\tan 37^\circ = \dfrac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ}$

$\cot 37^\circ = \dfrac{\cos 8^\circ + \sin 8^\circ}{\cos 8^\circ - \sin 8^\circ}$

04. $\dfrac{\cos 15^\circ - \sin 15^\circ}{\cos 15^\circ + \sin 15^\circ} = \dfrac{1}{\sqrt{3}}$

$\tan 30^\circ = \tan(45 - 15)$

$\dfrac{1}{\sqrt{3}} = \dfrac{\tan 45^\circ - \tan 15^\circ}{1 - \tan 45^\circ . \tan 15^\circ}$

$= \dfrac{1 + \tan 15^\circ}{1 - \tan 15^\circ}$

$= \dfrac{1 + \dfrac{\sin 15^\circ}{\cos 15^\circ}}{1 - \dfrac{\sin 15^\circ}{\cos 15^\circ}}$

$\dfrac{1}{\sqrt{3}} = \dfrac{\cos 15^\circ - \sin 15^\circ}{\cos 15^\circ - \sin 15^\circ}$

05. $\tan\left(\dfrac{\pi}{4} + \theta\right) - \tan\left(\dfrac{\pi}{4} - \theta\right) = \dfrac{4 \tan \theta}{1 - \tan^2\theta}$

LHS

$= \dfrac{\tan {}^{\pi}/_{4} + \tan\theta}{1 - \tan {}^{\pi}/_{4} . \tan\theta} - \dfrac{\tan {}^{\pi}/_{4} - \tan\theta}{1 + \tan {}^{\pi}/_{4} . \tan\theta}$

$= \dfrac{1 + \tan\theta}{1 - \tan\theta} - \dfrac{1 - \tan\theta}{1 + \tan\theta}$

$= \dfrac{(1 + \tan\theta)^2 - (1 - \tan\theta)^2}{(1 - \tan\theta).(1 + \tan\theta)}$

$= \dfrac{1 + 2\tan\theta + \tan^2\theta - (1 - 2\tan\theta + \tan^2\theta)}{1 - \tan^2\theta}$

$= \dfrac{1 + 2\tan\theta + \tan^2\theta - 1 + 2\tan\theta - \tan^2\theta}{1 - \tan^2\theta}$

$= \dfrac{4\tan\theta}{1 - \tan^2\theta}$

= RHS

06.

$\dfrac{1}{\tan 3\theta - \tan\theta} + \dfrac{1}{\cot\theta - \cot 3\theta} = \cot 2\theta$

LHS

$= \dfrac{1}{\tan 3\theta - \tan\theta} + \dfrac{1}{\cot\theta - \cot 3\theta}$

$= \dfrac{1}{\tan 3\theta - \tan\theta} + \dfrac{1}{\dfrac{1}{\tan\theta} - \dfrac{1}{\tan 3\theta}}$

$= \dfrac{1}{\tan 3\theta - \tan\theta} + \dfrac{\tan 3\theta . \tan\theta}{\tan 3\theta - \tan\theta}$

$= \dfrac{1 + \tan 3\theta . \tan\theta}{\tan 3\theta - \tan\theta}$

$= \dfrac{1}{\dfrac{\tan 3\theta - \tan\theta}{1 + \tan 3\theta . \tan\theta}}$

$= \dfrac{1}{\tan(3\theta - \theta)}$

$= \dfrac{1}{\tan 2\theta}$

$= \cot 2\theta$ = RHS

07.

tan A = 5⁄6 ; tan B = 1⁄11 ;

prove that : A + B = $^{\pi}/_{4}$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A . \tan B}$

$= \dfrac{\dfrac{5}{6} + \dfrac{1}{11}}{1 - \dfrac{5}{6} . \dfrac{1}{11}} = \dfrac{55 + 6}{66 - 5} = \dfrac{61}{61}$

$\tan(A + B) =$ **1**

$A + B = {}^{\pi}/_{4}$ …… PROVED

Q-6

01.

$\sin(25^\circ + \theta).\cos(25^\circ - \phi) - \cos(25^\circ + \theta).\sin(25^\circ - \phi) = \sin(\theta + \phi)$

LHS

$= \sin(25^\circ + \theta).\cos(25^\circ - \phi) - \cos(25^\circ + \theta).\sin(25^\circ - \phi)$

THIS IS : SIN A COS B – COS A SIN B = SIN (A – B)

$= \sin\left[(25^\circ + \theta) - (25^\circ - \phi)\right]$

$= \sin(25^\circ + \theta - 25^\circ + \phi)$

$= \sin(\theta + \phi)$

= RHS

02.

$\cos(35^\circ + A) . \cos(35^\circ - B) + \sin(35^\circ + A) .\sin(35^\circ - B) = \cos(A + B)$

LHS

$= \cos(35^\circ + A) . \cos(35^\circ - B) + \sin(35^\circ + A) .\sin(35^\circ - B)$

THIS IS : COS A COS B + SIN A SIN B = COS (A – B)

$= \cos\ (35^\circ + A) - (35^\circ - B)$

$= \cos\ (35^\circ + A - 35^\circ + B)$

$= \cos(A + B)$

= RHS

03.

$\sin\left(\frac{\pi}{6} + A\right) . \cos\left(\frac{\pi}{3} - B\right) + \sin\left(\frac{\pi}{3} - B\right) . \cos\left(\frac{\pi}{6} + A\right) = \cos(A - B)$

LHS : REARRANGING THE LAST TERM

$\sin\left(\frac{\pi}{6} + A\right) . \cos\left(\frac{\pi}{3} - B\right) + \cos\left(\frac{\pi}{6} + A\right) \sin\left(\frac{\pi}{3} - B\right) .$

THIS IS : SIN A COS B + COS A SIN B = SIN (A + B)

$= \sin\left(\frac{\pi}{6} + A + \frac{\pi}{3} - B\right)$

$= \sin\left(\pi/2 + A - B\right)$

$= \cos(A - B)$ .... $\sin(90 + \theta) = \cos\theta$

04.

$\cos\left(\frac{\pi}{6} - A\right) . \cos\left(\frac{\pi}{3} + B\right) - \sin\left(\frac{\pi}{6} - A\right) . \sin\left(\frac{\pi}{3} + B\right) = \sin(A - B)$

THIS IS : COS A COS B – SIN A SIN B = COS (A + B)

$= \cos\left(\frac{\pi}{3} - A + \frac{\pi}{6} + B\right)$

$= \cos\left(\pi/2 - A + B\right)$

$= \cos\left(\pi/2 - (A - B)\right)$

$= \sin(A - B)$ .... $\cos(90 - \theta) = \sin\theta$

Q-7

01. $\sin A = \frac{4}{5}$; $\frac{\pi}{2} < A < \pi$

$\cos B = \frac{5}{13}$; $\frac{3\pi}{2} < B < 2\pi$

Find

a) sin (A + B) b) cos (A – B)

c) tan (A – B)

A lies in the II Quadrant

∴ sin A & cosec A are positive

$\sin A = \frac{4}{5}$

$\sin^2 A + \cos^2 A = 1$

$\frac{16}{25} + \cos^2 A = 1$

$\cos^2 A = 1 - \frac{16}{25}$

$\cos^2 A = \frac{9}{25}$

$\cos A = -\frac{3}{5}$

$\tan A = -\frac{4}{3}$

B lies in the IV Quadrant

∴ cos B & sec B are positive

$\cos B = \frac{5}{13}$;

$\sin^2 B + \cos^2 B = 1$

$\sin^2 B + \frac{25}{169} = 1$

$\sin^2 B = 1 - \frac{25}{169}$

$\sin^2 B = \frac{144}{169}$

$\sin B = -\frac{12}{13}$

$\tan B = -\frac{12}{5}$

Now

$\sin A = \frac{4}{5}$; $\cos A = -\frac{3}{5}$; $\tan A = -\frac{4}{3}$

$\sin B = \frac{-12}{13}$; $\cos B = \frac{5}{13}$; $\tan B = -\frac{12}{5}$

a) sin (A + B)

$= \sin A . \cos B + \cos A . \sin B$

$= \frac{4}{5} . \frac{5}{13} + -\frac{3}{5} . -\frac{12}{13}$

$= \frac{20}{65} + \frac{36}{65}$

$= \frac{56}{65}$

b) cos (A – B)

$= \cos A . \cos B + \sin A . \sin B$

$= \frac{-3}{5} . \frac{5}{13} + \frac{4}{5} . \frac{-12}{13}$

$= -\frac{15}{65} + \frac{-48}{65}$

$= \frac{-15 - 48}{65}$

$= -\frac{63}{65}$

c) tan (A – B)

$= \frac{\tan A - \tan B}{1 + \tan A . \tan B}$

$= \frac{-\frac{4}{3} - \left(\frac{-12}{5}\right)}{1 + \frac{-4}{3} . \frac{-12}{5}}$

$= \frac{-\frac{4}{3} + \frac{12}{5}}{1 + \frac{48}{15}}$

$= \frac{\frac{-20 + 36}{15}}{\frac{15 + 48}{15}} = \frac{16}{63}$